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LETTER TO THE EDITORS

A REDUCED FORMULA FOR THE DYNAMIC-BIAS IN RADIATION INTERROGATION OF TWO-PHASE FLOW

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IN A previous analysis concerning the dynamic-bias which appears in radiation interrogation of two-phase flow [1], an expression for the bias was derived and given in the form

$$\Delta\alpha = \frac{1}{\lambda}\ln\left\{\frac{1}{\lambda}\int_0^\tau e^{\lambda\alpha(t)}\,dt\right\} - \frac{1}{\tau}\int_0^\tau \alpha(t)\,dt. \qquad (1)$$

where

$\alpha(t)$ = time dependent void fraction along the direction of radiation transmission,
λ = total channel thickness in units of mean-free-path of the transmitting radiation,
τ = time interval of measurement.

It has become apparent that the formula for this dynamic-bias, equation (1), can be expressed in alternative form which is algebraically simpler and permits a more intuitive appreciation of the dynamic-bias.

We write the last term of equation (1) as

$$\frac{1}{\tau}\int_0^\tau \alpha(t)\,dt = \langle\alpha\rangle, \qquad (2)$$

or, equivalently

$$\frac{1}{\tau}\int_0^\tau \alpha(t)\,dt = -\frac{1}{\lambda}\ln\{e^{-\lambda\langle\alpha\rangle}\}. \qquad (3)$$

Substitution of equation (3) into equation (1) yields

$$\Delta\alpha = \frac{1}{\lambda}\ln\left\{\frac{1}{\tau}\int_0^\tau e^{\lambda\alpha(t)}\,dt\right\} + \frac{1}{\lambda}\ln\{e^{-\lambda\langle\alpha\rangle}\}. \qquad (4)$$

Following some algebraic rearrangement we obtain

$$\Delta\alpha = \frac{1}{\lambda}\ln\left\{\frac{1}{\tau}\int_0^\tau e^{\lambda[\alpha(t)-\langle\alpha\rangle]}\,dt\right\}. \qquad (5)$$

Here we note the importance of the deviation of $\alpha(t)$ from the mean $\langle\alpha\rangle$ and the several operations performed thereon. Expansion of the exponential term in an algebraic series leads to

$$\Delta\alpha = \frac{1}{\lambda}\ln\left\{\frac{1}{\tau}\int_0^\tau \sum_{n=0}^\infty\left[\frac{\lambda^n[\alpha(t)-\langle\alpha\rangle]^n}{n!}\right]\right\}dt,$$
$$= \frac{1}{\lambda}\ln\left\{1 + \sum_{n=2}^\infty \frac{\lambda^n}{n!}\int_0^\tau [\alpha(t)-\alpha(t)]^n\,dt\right\}. \qquad (6)$$

That is

$$\Delta\alpha = \frac{1}{\lambda}\ln\{1 + \text{higher moments of }[\alpha(t)-\langle\alpha\rangle]\}. \qquad (7)$$

Hence, for a given channel thickness, fluid median, and radiation source, the dynamic-bias is governed entirely by the higher moments of the void fraction $\alpha(t)$ about its mean $\langle\alpha\rangle$.

REFERENCES

1. A. A. Harms and F. A. R. Laratta, The dynamic-bias in radiation interrogation of two-phase flow, *Int. J. Heat Mass Transfer* **16**, 1459 (1973).

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ERRATUM

TERUKAZU OTA, A Riemann–Hilbert problem for a heat condition in an infinite plate with a rectangular hole, *Int. J. Heat Mass Transfer* **16**, 1941–1943 (1973).

In the 7th line from the bottom of the right column of page 1941, Φ is missing ahead of (Z). In equation (9) on page 1942, a minus sign $(-)$ should be inserted ahead of the third term of C_0. In the 12th line of the right column of page 1943, $q_{y0}(\zeta')$ is to be replaced with $q_{y0}(\xi')$.